**ECOM90024**

**FORECASTING IN ECONOMICS AND BUSINESS**

**PRACTICE EXAM SOLUTIONS**

**SEMESTER 1, 2023**

**NOTE: PLEASE COMPUTE ALL OF YOUR NUMERICAL ANSWERS TO 3 DECIMAL PLACES**

**Question 1 (11 Marks)**

Let be governed by the following process,

The parameters , , , are unknown but you have observed a set of realizations .

1. **(5 Marks)** Outline the steps that you would take in order to test whether there exists a unit root in the data generating process of . Be as explicit and precise as possible.

**To test for a unit root, we first want to add and subtract to the right hand side equation governing**

**Then we subtract from both sides to obtain**

**If there exists a unit root in , it must be the case that . Therefore, we can estimate the following linear regression**

**And then test the null hypothesis that against the alternative that it is not equal to zero. The test statistic is computed as the standard test statistic**

**We would compare this test statistic to the set of critical values from the Dickey Fuller test table associated with an equation that includes a mean and trend. If the test statistic lies in the rejection region, then we would reject the null hypothesis and conclude that the data generating process is covariance stationary**.

1. **(2 Marks)** Using words, explain how your forecast intervals of will depend on the outcome of the procedure that you outlined in part (a).

**Whenever we perform an augmented Dickey-Fuller (ADF) test, there are two possible outcomes:**

1. **There exists a unit root in the data generating process.**

**In this scenario, the forecast error variance will be an increasing function of the forecast horizon and will increase without bound. This will mean that the forecast intervals will widen without bound as we increase the forecast horizon.**

1. **There does not exist a unit root in the data generating process.**

**In this scenario, the forecast error variance will converge to the unconditional variance of as the forecast horizon . This will mean that the forecast intervals will be bounded as we increase the forecast horizon.**

1. **(4 Marks)** Assume that is trend stationary. Describe two appropriate approaches to estimating the parameters and . Be as explicit and precise as possible.

**The first approach to estimating the parameters and would be the method of least squares. This approach computes the estimates as the values that minimise the sum of squared errors. That is, given a set of observations , we would find the values that solve the following objective function:**

**The second approach to estimating the parameters and would be the method of maximum likelihood. This approach computes the estimates as the values that maximise the conditional log-likelihood function. Since the innovations are specified to be Gaussian with mean zero and a variance of , the conditional likelihood function is given by**

**That is, given a set of observations , we would find the values that solve the following objective function:**

**The third approach would be to de-trend the data using OLS and then using the Yule-Walker equations to compute estimates of and . That is, we would first estimate the following regression,**

**This will produce a set of residuals . From these residuals we would compute the first two sample autocorrelations as**

**Then with these we can compute the Yule-Walker estimates as**

**Question 2 (10 Marks)**

Let be a time series that behaves according to the following equation,

1. **(2 Marks)** Compute the unconditional mean and variance of .

**The unconditional mean and variance are computed as**

**(1 Mark)**

**(1 Mark)**

1. **(2 Marks)** Is a covariance stationary process? Please make sure to include your reasoning and any derivations or computations that you have employed.

**Yes, is an MA(2) process in which the moving average coefficients are real and finite. Therefore, it will have a constant mean and variance and it’s autocovariances will be time-invariant. (2 Marks)**

1. **(2 Marks)** Is an invertible process? Please make sure to include your reasoning and any derivations or computations that you have employed.

**Writing the model in terms of lag operators, we see that**

**(1 Mark)**

**Therefore the lag polynomial can be expressed as**

**The roots of this quadratic equation are clearly greater than 1. Therefore we can conclude that this MA(2) process is invertible. (1 Mark)**

1. **(4 Marks)** Derive the autocorrelation function of and provide an appropriate visual depiction of it.

**We know that the autocorrelation function for an MA(2) is zero for , therefore we only need compute the first and second autocorrelations. First we compute the autocovariances**

**Simplification yields,**

**(1 Mark)**

**(1 Mark)**

**The autocorrelations are then computed by dividing these two quantities through by the variance, (1 Mark)**

**Providing an appropriate sketch of the ACF gives (1 Mark).**

**Question 3 (17 Marks)**

Suppose that you have generated the following sample ACF and PACF from a set of time series observations,



1. **(4 Marks)** Using the visual features depicted in the sample ACF and PACF and your knowledge of the stochastic properties of autoregressive and moving average models, argue that an AR(2) would be a more appropriate specification compared to an MA(2).

**Looking at the ACF we can see that the sample autocorrelations are statistically significant and gradually decaying as becomes large. This is consistent with the dependence structure of an autoregressive model and inconsistent with the dependence structure of a moving average model of order 2. The latter model would have an ACF function that is zero for . (2 Marks)**

**Looking at the PACF we can see that the sample partial autocorrelations are no longer statistically different from zero for . This is consistent with the dependence structure of an autoregressive model and inconsistent with the dependence structure of a moving average model. (2 Marks)**

1. **(4 Marks)** Suppose that the sample autocorrelations are computed as,

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 0.824 | 0.752 | 0.660 | 0.573 | 0.522 | 0.446 | 0.388 | 0.348 | 0.282 |

Assuming that the true data generating process is an AR(2), write down the expressions for the first two autocorrelations in terms of the AR(2) coefficients and and verify that their Yule-Walker estimates are given by and

**The Yule-Walker estimates are computed from the equations**

**(2 Marks)**

**We can rewrite these equations in terms of the AR coefficients**

**Substituting in for we are able to express in terms of the autocorrelations**

**So that**

**Therefore,**

**(1 Mark)**

**(1 Mark)**

1. **(2 Marks)** If the data generating process is an AR(2), its variance will be given by,

Now suppose that the sample variance of your time series is computed to be . Using this and the quantities that you verified in part b, verify that the estimate of the innovation variance is given by .

**We can compute the estimate of the innovation variance by solving for and plugging in the estimates**

**(2 Marks)**

1. **(6 Marks)** Assume that the data generating process has an unconditional mean of 0 and suppose that the last two observations in the sample take values and . Using the quantities that you obtained in part b.) and c.), compute 1-step and 2-step ahead point and 95% interval forecasts.

**The point forecasts are given by**

**(1 Mark)**

**(1 Mark)**

**To compute the interval forecasts, we first recognize that the forecast errors will be given by**

**Therefore the forecast error variances will be**

**(1 Mark)**

**(1 Mark)**

**Therefore the 95% forecast intervals will be given by**

**(1 Mark)**

**(1 Mark)**

1. **(1 Mark)** Briefly describe in words what will happen to the forecast interval as the forecast horizon becomes arbitrarily large, (i.e. ).

**The forecast interval will grow as but it will be bounded. This is because the forecast error of a covariance stationary time series is itself a covariance stationary series and it’s variance will always be bounded by the square-summability of the coefficients from its Wold representation. (1 Mark)**